

Exercise 9.3

$$L = [y^1 - y^0 \quad y^2 - y^0 \quad \dots \quad y^n - y^0]^T$$

$$= \begin{bmatrix} (y^1 - y^0)^T \\ \vdots \\ (y^n - y^0)^T \end{bmatrix} = \begin{bmatrix} y_1^1 - y_1^0 & \dots & y_n^1 - y_n^0 \\ \vdots & & \vdots \\ y_1^n - y_1^0 & \dots & y_n^n - y_n^0 \end{bmatrix}$$

$$\hat{L} = \frac{1}{\Delta} L = \frac{1}{\Delta} [y^1 - y^0 \quad \dots \quad y^n - y^0]^T$$

$$= \begin{bmatrix} \frac{y_1^1 - y_1^0}{\Delta} & \dots & \frac{y_n^1 - y_n^0}{\Delta} \\ \vdots & & \vdots \\ \frac{y_1^n - y_1^0}{\Delta} & \dots & \frac{y_n^n - y_n^0}{\Delta} \end{bmatrix}$$

$$L_\gamma = [y_\gamma^1 - y_\gamma^0 \quad y_\gamma^2 - y_\gamma^0 \quad \dots \quad y_\gamma^n - y_\gamma^0]^T$$

$$= \begin{bmatrix} (y_\gamma^1 - y_\gamma^0)^T \\ (y_\gamma^2 - y_\gamma^0)^T \\ \vdots \\ (y_\gamma^n - y_\gamma^0)^T \end{bmatrix} = \begin{bmatrix} y_{\gamma+n}^1 - y_{\gamma+n}^0 \\ \vdots \\ y_{\gamma+n}^n - y_{\gamma+n}^0 \end{bmatrix}$$

$$\hat{L}_\gamma = \frac{1}{\Delta_\gamma} L_\gamma = \frac{1}{\Delta_\gamma} [y_\gamma^1 - y_\gamma^0 \quad y_\gamma^2 - y_\gamma^0 \quad \dots \quad y_\gamma^n - y_\gamma^0]^T$$

^,

$$= \begin{bmatrix} \frac{y_1' - y_1^0}{\Delta} & \dots & \frac{y_n' - y_n^0}{\Delta} \\ \vdots & & \vdots \\ \frac{y_1'' - y_1^0}{\Delta} & \dots & \frac{y_n'' - y_n^0}{\Delta} \end{bmatrix}$$

Hence $\frac{1}{\Delta} L = \frac{1}{\Delta \gamma} L \gamma$